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Building Depth of Understanding of the Concept of Number Sense and Algebra in Students Grades 7 and 8

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Building Depth of Understanding of the Concept of Number Sense and Algebra in Students Grades 7 and 8

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Signature Sheet

This thesis, written under the direction of the candidate's thesis advisor and approved by the Chair of the Master's program, has been presented to and accepted by the Faculty of Education in partial fulfillment of the requirements for the degree of Master of Science. The content and research methodologies presented in this work represent the work of the candidate alone.

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Abstract

Mathematics instruction offers both challenges and rewards to students. Teachers tend to focus on transmission of knowledge. Student creativity and problem-solving skills are not typically activated with common teaching practices. When students are engaged in mathematics they are recognizing patterns, making connections on their own, coming up with creative solutions, and discussing mathematical ideas with peers and adults. The purpose of this study was to identify, employ, and evaluate strategies that engage students in these practices and assist them in developing a deep understanding of mathematics concepts such as number sense and algebra.

The review of the literature began with the history of mathematics instruction. The research reviewed alternative methods in mathematics education that engage students in mathematical practices. The research concerning cognitive development indicated that these strategies are successful in teaching mathematics to adolescents.

This was a participatory teacher action research study. The participants were groups of fifteen 7th and 8th grade students from lower- and middle-class families. Student pre- and post-test scores on a standard chapter test on number sense and algebra were compared following teaching practices involving teacher-student and student-student interaction. Students asked each other questions about the material and verbally justified their methods. Data along with student and teacher observations were analyzed to evaluate the effectiveness of those strategies on the participants' understanding of mathematics. Results indicated that there was an increase in student understanding of number sense and algebra based on pre- and post-test score comparisons and student and teacher narrative.

Chapter 1 Building Understanding of Mathematics

Lockhart (2009) in *A Mathematician's Lament* begins with the analogy of comparing today's era, with its specific ideas about mathematics education, to a world in which music is mandatory. Alluding to the idea that true mathematics is not taught in schools post high school, he writes, "playing and listening to music, let alone composing an original piece, are considered very advanced topics and are generally put off until college, and more often graduate school" (Lockhart, 2009, p. 16). Mathematics, a subject that he and other mathematicians found essentially beautiful has been twisted into something painful and rote (Lockhart, 2009). This idea sets the stage for Lockhart's central thesis: mathematics is beautiful and exciting, an art form that should not be tortured and modified, but upheld in all its purity.

Lockhart (2009) argues that as students develop an appreciation for the true beauty of mathematics, they will fall in love with it. In traditional educational methods, students hate something that is not purely mathematics, but is rather disguised as mathematics (Lockhart, 2009). The intention of the study was to explore alternative methods of teaching that allow students to be engaged in mathematics by interacting with their peers and the material as well as asking probing questions in order to construct their own understanding.

Statement of Problem

In the history of mathematics education in the United States, there is a tradition of passing on knowledge of algorithms through lectures followed by independent practice. There is hope for a shift in instructional methods with the introduction of Common Core State Standards (CCSS), which call for deeper thinking about mathematics concepts than the previous standards elicited (Common Core State Standards Initiative, 2015). However, lack of professional development that focuses on effective and appropriate strategies has caused the transition to be difficult. Mathematics educators in middle school classrooms continue to focus on the transmission of knowledge rather than engaging students in interactive dialoging and problem solving.

Student creativity and problem-solving skills are not activated with common teaching practices that have been and continue to be employed in mathematics classrooms (Center for the Study of Mathematics Curriculum, 2004; Ellis & Berry, 2005). In mathematics classrooms, students should be engaged in dialogue with peers and the teacher. They should be constructing their understanding of the concepts through difficult problem-solving. Students have not mastered a subject if they can only recite formulas and apply algorithms. If students have truly mastered mathematics, then they would be able to solve unique problems in new situations (Center for the Study of Mathematics Curriculum, 2004; Common Core State Standards Initiative, 2015)

Not only do students have difficulty mastering mathematics concepts, but they also feel unsuccessful in their mathematical ability. Students are not excited about mathematics and they do not feel successful at it (Yavuz Mumcu & Cansiz, 2015). Some mathematics students in the US are disengaged and disinterested. Engagement in the classroom directly correlates to student academic success (Brown, 2009). Due to the disengagement that teachers are seeing in mathematics classrooms, it is important for educators and researchers to search for effective instructional methods.

Purpose Statement

The purpose of this study was to test the effectiveness of student interaction with their teachers, peers, and mathematical materials as an instructional method for deepening student understanding in number sense and algebra. When students are interacting with one another, they are able to reach complex ideas more easily and quickly (Goos, Galbraith, & Renshaw, 2002). The teacher also benefits from dialogue with their mathematics students; teachers are able conduct formative assessments of student understanding as well as correct and deepen student understanding (Goos, 2004). The teaching methods that were employed in this study were selected based on developmental appropriateness for adolescents.

Research Question

How does increased social interaction and dialogue with other students and the instructor increase student understanding of number sense and algebra? Answering this question assists educators in their transition to the CCSS. This research adds to the literature concerning best practices in mathematics instruction for adolescents.

For the purposes of this study, the following terms were defined as:

Social interaction – students discussing with or asking questions of other students and their teacher about the math material.

Dialogue – students and teachers discussing math concepts with the goal of constructing their understanding. “So, dialogue presupposes curiosity; it doesn't exist without epistemological curiosity, without the desire to understand the world around us. That is what differentiates dialogue from simple conversation. Such curiosity embodies the conscious willingness to engage in a search for the meaning of an object, to clarify or apprehend the full meaning” (Leistyna, 2004. p. 19).

Increased student understanding – students understanding on a deep level—students who have increased understanding demonstrate the ability to problem solve, ask questions, look for patterns, and apply previously-taught skills.

Number sense – an understanding of the relationship of numbers to one another as well as fluidity with simple operations

Algebra – a branch of mathematics in which letters are used to represent numbers in expression and equations

Constructivist approach – In the constructivist approach (Southwest Educational Development Laboratory (SEDL), 1994) students interact with adults, peers, and models to build their understanding of a topic. Students engage in authentic learning tasks. Students explore ideas, ask questions, and reflect on their learning.

Theoretical Rationale

The research topic was based on the theory of constructivism. Jonassen, as cited in Chen (n.d.), discusses a few characteristics of constructivist theory. When constructivism is used in the classroom, students are encouraged to view the world as a complex place. They are immersed in meaningful and authentic tasks that help them to construct their own knowledge rather than receive it from the teacher. Constructivist environments encourage metacognition and reflection, which allows students to not only to discover the content, but also to discover how they best learn and access information. A constructivist classroom is a collaborative environment in which students are interacting with one another to discover knowledge while the teacher is acting as their guide. There are two strands of constructivism: cognitive constructivism and social constructivism.

Cognitive constructivism is based upon Piaget's theory (as cited in Jonassen, 1994) that humans cannot immediately understand information that is told to them, but that they must build their understanding through experience. Cognitive constructivism calls for the teacher to be a guide and lead students through their own construction of their understanding. In social constructivism, however, the teacher plays an active role by interacting with the students during challenging tasks compared to teachers in a traditional setting.

Social constructivism is based on the theories of Vygotsky (as cited in Jonassen, 1994) and is founded upon the idea that students learn best when they are in their zone of proximal development (ZPD). A task that an individual cannot complete alone but can complete with others is a task that is in their ZPD. Chen (n.d.) quotes Maddux, Claiborne, Johnston and Willis who discuss the ZPD:

> Vygotsky's ZPD emphasizes his belief that learning is, fundamentally, a socially mediated activity. Thinking and problem-solving skills can, according to Vygotsky, be placed in three categories. Some can be performed independently by the child. Others cannot be performed even with help. Between these two extremes are skills the child can perform with help from others. Those skills are in the ZPD. If a child uses these cognitive processes with help of others, such as teachers, parents, and fellow students, they will develop skills that can be independently practices. As Vygotsky (1987) puts it, “What the child is able to do in collaboration today he will be able to do independently tomorrow (para. 1).

Students may not succeed when working on a difficult task alone, but when they explore and interact with peers and adults, they can be successful. In the future, they may be able to demonstrate their ability to perform a task alone that was previously in their ZPD.

The researcher recognized the importance of cognitive constructivism, but focused on social constructivism because of the active role of the teacher. Student experience with mathematics instruction can be difficult, therefore students need to interact and speak about math that is in their ZPD. “Social discourse helps students change or reinforce their ideas. If they have the chance to present what they think and hear others' ideas, students can build a personal knowledge base that they understand.” (SEDL, 1994, para. 14). Interacting with peers and the teacher is a vital part of students’ ability to gain access to more complex tasks. In order to have success with a social constructivist in the classroom, the teacher must create a safe environment for students to interact, take risks, and explore.

Assumptions

The assumptions were that the traditional methods that transmit knowledge from teacher to student are not the ideal methods for teaching mathematics. The assumptions at the start of this research were that students learn best when they are actively engaged in their learning with peers and adults; students need the opportunity to construct their own knowledge and build neuro pathways that will benefit them as they reach adulthood.

Background and Need

Lau, Singh, and Hwa (2009) describe the importance of social interaction within the context of a constructivist classroom. The focus of this study was to transition a teacher to a student-centered, interactive classroom and develop a four-step lesson plan. The researcher developed a lesson plan format based on the social aspects of the lesson, namely, the interactions between the teacher and students. The problem that the author identifies is that the “math skills [students need] to function in the workplace” have changed (Lau et al., 2009, p. 307). Employers demand more of students when they become a part of the workforce in terms of problem solving, but teachers need to prepare them for that with something better than the traditional methods of delivering knowledge. The purpose of the study by Lau et al. (2009) was to develop a 4-step lesson plan and create an interactive classroom setting for a teacher's classroom. The other purpose was to discover the types of interaction that helped students to improve their mastery of concepts. The research questions were "How is a four-phase lesson plan that blends whole class and small group interactions developed to promote interactive classroom context?" and "What are the different types of teacher-student interaction that can enhance students' learning in the classroom?" (Lau et al., 2009, p. 308).

The participants consisted of thirty 16- and 17-year-olds and their math teacher with 15 years of experience from Samarahan School in the State of Sarawak, Malaysia. The researchers studied 58 lessons through audio recordings, videotapes, written observations, and interviews with the teacher and with the students. The validity of this study was created through data triangulation: multiple instruments were used to collect data. Qualitative data were collected through recordings, observations, and interviews. The information was organized and presented through segments of the lessons that highlighted key interactions between teacher and student or student and student. The teacher was asked to reflect on these lessons and interpret what was happening in the lesson. The students were interviewed about how they felt about learning math.

The teacher-student interactions were sorted into categories: student need, student suggestion, evaluation, and justification; the student-student interactions were named either thinking, communicating, or off-task talk. The type of activity was categorized and recorded during data analysis. The type of activity was also categorized as either challenging, related to prior knowledge, or promoting discussion. This analysis helped the teacher, with the help of the researchers, create the four-phase lesson plan that was carried out in subsequent lessons, making it an action-research study. The creation of this four-phase lesson plan was successful (Lau et al., 2009).

The key findings were that the teacher realized the benefits of the constructivist approach to mathematics and became increasingly willing to incorporate constructivist activities into his lesson. The teacher and researcher also became aware of the interactions that most improved students' understandings of the mathematics. These included the teacher inviting the student to participate in mathematical thinking and evaluating a student response or idea. The students demonstrated a deeper understanding of mathematics and expressed appreciation of this way of understanding (Lau et al., 2009).

Lau et al. (2009) helped to connect philosophies about teaching math with existing education theory of social constructivism. Students benefit from a chance to work in their ZPD by interacting with the teacher and their peers to develop an understanding of mathematics concepts. This self-discovery gives them increased ownership of the topic and the concepts. Lau et al. (2009) identified strategies and theories that helped to shape the direction of the research.

Summary

In order for students to appreciate and understand math, they need to experience it. This means that they should be working within their zone of proximal development with peers and the teacher to construct their own understanding of mathematics topics. Social constructivism provides a theoretical base on which educators can build instructional practices to better serve their students in math instruction. The study by Lau et al., 2009 followed a teacher who was able to create a lesson plan format that would guide his students through building their understanding of math content. This lesson plan format was successful not only because of its structure, but because there was a classroom culture that promoted interaction between peers and the teacher. This concept of social interaction and dialogue gave direction to this teacher action research study. The study extends the research on the effect of social interaction on student understanding of mathematics.

The following chapter is a review of the literature on the history of the development of standards surrounding mathematics, successful alternative methods in mathematics education that engage students in mathematics practices, and the cognitive and social development of adolescents.

Chapter 2 Review of the Literature

Introduction

This section is an examination of the peer-reviewed research literature on the history of the development of standards surrounding mathematics, successful alternative methods in mathematics education that engage students in mathematics practices, and the cognitive and social development of adolescents. Information was gathered from academic library searches using online resources. Research information was organized in the following categories: Historical Context and Review of the Academic Research

Historical Context

History of Mathematics Education.

Ellis and Berry (2005) explored the history of mathematics education. “Revisions” in mathematics education are defined as changes or tweaks in education that do not fundamentally solve the problems in mathematics education. “Reforms,” on the other hand, change how mathematics is thought about and “raise questions about the core beliefs of mathematics education […], how it is taught, how it is learned, and, ultimately, what constitutes success in learning it” (p. 8) According to the researchers, many mathematics education movements have not dramatically or successfully changed the way mathematics education is thought about and taught. At the beginning of the 20th century, Edward L. Thorndike, president of the American Psychological Association, put forth research concerning mathematics education that is still with us today. The method of mathematics education that Thorndike recommended is what we call the traditional method of mathematics education. He believed that mathematics is best learned by students when it is “explicitly taught, and then practiced with much repetition” (Ellis & Berry, 2005, p. 8). While criticized by some, this method spread and was widely accepted by mathematics educators. Critics argued that it did not encourage students to think critically when solving problems and did not take into account “the experiences the students bring to mathematics or the meaning they make of what is learned” (Ellis & Berry, 2005, p. 8) the way that a constructivist learning environment would.

The progressive movement, led by the Progressive Education Association, went against Thorndike’s idea. They were driven by the idea that students need connect the learning to their experiences and interests. Their theories and vision aligns with constructivist ideas. However, these ideas were seen as radical by many educators and the PEA had little impact on changes in mathematics education. Throughout the century, these reformist ideas continued to arise, but none became established practice. New programs were introduced, but they were full of flaws and were eventually discarded. When they were discarded, Thorndike’s ideas once again arose. Social efficacy, a product of the progressive movement, did catch on. After all of the changes, mathematics education had come full circle and looked very similar to how it began.

> Despite a century of ‘reform’ efforts, school mathematics practices in the late twentieth century remained stubbornly similar to what Florian Cajori […] described one hundred years earlier in his study of mathematics classrooms across the United States: ‘[There were] no explanations of processes either by master or pupil…the problems were solved, the answers obtained, the solutions copied’ (p. 10) […Teachers] were still the center of authority, first disseminating rote skills and procedural knowledge to their students who then worked individually on sets of problems in order to internalize this knowledge (Ellis & Berry, 2005, p. 10).

In the 1980s, researchers began looking for best practices for students and trying to identify how students master mathematical concepts effectively. Researchers referenced ideas from mathematics theory and cognitive development and began to believe that all students could learn mathematics, and that they would master concepts best if they were connected to student experiences. Suddenly, teachers were asking students to justify their answer by verbalizing and/or writing their explanations. Teachers were asking them to grapple with concepts and apply those concepts to new situations (Ellis & Berry, 2005). These new ideas surrounding mathematics education led to the development of national mathematics standards.

One pattern that the researcher noted in Ellis and Berry (2005) is that revisions were made to mathematics every time that a political or economic “crisis” emerged in the United States. The National Science Foundation (NSF) was developed in a reaction to the political and economic standing of the US in the world as a result of poor mathematics and science education. Russia’s launching of Sputnik in the space race led for a call for change in how teachers taught mathematics and science in our schools (Ellis & Berry, 2005). The US and NSF teamed up and created the School Math Study Group whose purpose was to produce curriculum that could be distributed quickly. This rushed project led to a mathematics curriculum that was abstract and not grounded in real-world problems (Ellis & Berry, 2005).

Today, the creation of the CCSS are a reaction US economic standing in the world and on global student comparisons of student achievement. Results collected by groups such as Program for International Student Assessment (PISA) (Organisation for Economic Co-operation and Development (OECD), 2015a) concern educators and political leaders when US student scores are low compared to student scores from other countries. The fields of science and mathematics are important in developing new technology. Therefore, political leaders, economists, and the general public are concerned with how US students are performing on standards-based mathematics tests. PISA assessment results are not just reporting on student success in mathematics. They also carry the added implication that the US is behind in its global economic and political standing. It is important to look more closely at the mathematical proficiency of both teachers and students in the US.

Student and Teacher Mathematical Competence.

Both student and teacher understanding of mathematics in the US is below that of other countries (OECD, 2015b). The PISA results indicate that students from the US have less mathematical understanding when compared with other countries. “The United States performed below average in 2012 and is ranked 27th” (OECD, 2015b, p. 1). The PISA website comments on the importance of mathematical competence and explains what it means for a student to have depth of understanding in mathematics. “A mathematically literate student recognises the role that mathematics plays in the world in order to make well-founded judgments and decisions needed by constructive, engaged and reflective citizens” (OECD, 2015a, para. 2). To be considered mathematically competent, a student needs to be able to apply mathematical knowledge in new contexts. The quote from PISA indicates just how important it is for a nation to foster this ability in their citizens.

The US Department of Education understands the definition and importance of mathematical literacy and recognizes the need for improvement in mathematics education.

> International tests show that the United States is, at best, running in place, while other nations are passing us by. Many countries now match or exceed us, not only in the number of years their children attend school but also in how much those

> children learn. The United States was once the world leader in high school completion, but among our 25–34 year olds, it has now slipped to 10th place, falling behind such countries as Canada, Switzerland, and South Korea. It may fall farther behind yet. The same is true for achievement. On most international tests, the United States is standing still while others are gaining ground (U.S. Department of Education, 2008, p. 9).

The rankings of the US students in mathematics indicate that the US is behind other countries. PISA further analyzed the US test results and found that US students did well on more simple and straightforward tasks such as gathering data from tables and using formulas. On the other hand, students showed a weakness in mathematical reasoning. "Students in the United States have particular weaknesses in performing mathematics tasks with higher cognitive demands, such as taking real-world situations, translating them into mathematical terms, and interpreting mathematical aspects in real-world problems" (OECD, 2015, p. 1). Student mathematical competence is still functioning at a low level. From the researcher's perspective, student performance and teacher competence are directly related.

An excellent mathematics teacher has confidence in mathematics. Unfortunately, most teachers in elementary and secondary education are unprepared to teach mathematics (Beckmann, 2010). Teachers sometimes are resistant to teaching mathematics because they are inadequately prepared due to their own poor mathematics education (Steele, Brew, Rees, & Ibrahim-Khan, 2013). It is unrealistic to expect teachers to instill in their students a love and understanding of mathematics when they themselves shy away from the subject. In order for students to feel more competent in mathematics, they must be taught by a teacher who themselves is competent in mathematics. In the researcher's opinion, this implies a need for more teacher training in mathematics.

Though there have been some revisions to mathematics education in the past century, there have been no major reforms. In order for students to demonstrate mathematical competence on exams such as PISA, effective teaching strategies must be employed in the classroom. The following section explores the existing literature concerning appropriate and effective strategies for middle school students in the mathematics classroom.

Review of Academic Research

Brain Research Adolescent Development.

Human brains are impressive because of their plasticity, or their ability to continually change and adapt to new situations, even into late adulthood. While they never stop changing, much growth and dramatic change in development happens during adolescence (Sercombe & Paus, 2009). During adolescence, the brain develops from back to front. Specifically, this means that the amygdala, the fear and emotional center of the brain, develops before the prefrontal cortex, the reasoning center of the brain (Casey, Tottenham, Liston, & Durston, 2005).

The brain is composed up of a massive network of neurons, or brain cells (Sercombe & Paus, 2009). These neural connections determine our patterns of thought. These neural connections are collectively referred to as gray matter, the "roads" that our thoughts take. During childhood, learning is happening fast and many connections ("roads") are being made. Research described that in early adolescence, the volume of gray matter peaks and then begins to decrease (Sercombe & Paus, 2009; Casey et al., 2005; Choudhury, Charman, & Blakemore, 2008). At the same time, white matter develops rapidly to strengthen and solidify the connections that are still present, making the little dirt “roads” into “highways” (Sercombe & Paus, 2009; Casey et al., 2005; Choudhury et al., 2008).

The development of the brain cannot be separated from the environment in which it is placed. The connections made and solidified are partially influenced by the social environment that is part of the individual experience. The purpose of the literature review conducted by Sercombe and Paus (2009) was to discuss the neuroscience research and technology in recent years and then explore what this means for teachers’ understanding of and interactions with teens. They discussed how understanding the tight connection between cognitive development and social development can help shape teaching practices as educators (Sercombe & Paus, 2009).

Social and cognitive development research supported that adolescence is an emotional time. The cognitive changes that happen at this time are greatly influenced by the social environment (Sercombe & Paus, 2009). During adolescence, there is a peak in negative emotions and adolescents’ emotional responses to situations are extreme (Pöhland & Raufelder, 2014). The emotional outputs documented in the research could be explained by understanding cognitive development. “One possible explanation for this [increase in extreme emotions] could be a decreased influence of the frontal cortex, leading to increased activity in the limbic system (e.g. amygdala)” (Pöhland & Raufelder, 2014, p. 452). As previously stated, the brain is developing from back to front, with the prefrontal cortex, the reasoning center, the last to develop. The researched proposed that the amygdala, the emotional center, sometimes takes over in situations where the prefrontal cortex would typically be used. In addition to increasing one’s understanding the influences of the different brain regions, it is also important to note the sequential development of the brain, as teachers think about their role as educators.

Sercombe and Paus (2009) described the growth of the brain during the teen years and summarized implications of this research upon teaching practice as influential adults in teens' lives. The amount of white matter growth during adolescence implied that this is the time when the neuropathways are solidified. Specifically, this means that if teens do or think about something one way, they will most likely choose that same "easy" path in the future (Sercombe & Paus, 2009). In addition, Casey et al. (2009) draws attention to the decrease in the production of gray matter. Pruning occurs in the prefrontal cortex during adolescence (Casey et al., 2009).

Sercombe and Paus (2009) cautioned their readers not to oversimplify the complex workings of the brain and the influences from both genes and the environment. On the other hand, one must consider findings from research studies that document the cognitive changes and reactions that result from the environment (Pöhland & Raufelder, 2014; Masten, Telzer, Fuligni, Lieberman, & Eisenberger, 2012). As stated in Choudury et al. (2008), "it is unknown whether the pruning of synapse in frontal cortex during adolescence is similarly influenced by the environment. If this turns out to be the case, it would have profound implications for the kinds of experiences and environments that are optimal for teenage brain development" (p. 146). Teachers need to consider how our actions as educators, including the way that we allow students to interact in our classrooms, may strongly impact students' cognitive development.

Developmentally Appropriate Practices.

Meschke, Peter, and Bartholomae (2012) use brain research to describe what is occurring developmentally for children in early adolescence. Researchers addressed using strategies that can be effective teaching strategies with this age group. They posit that teens need a safe space to take risks and practice their ability to publicly express themselves. Ideally, this can occur in a whole class setting. However, starting with small collaborative groups gives students a chance to try out their ideas in a more intimate setting. "Small group activities may be an effective introduction to reduce the adolescents' self-consciousness and intimidation" (Meschke et al., 2012, p. 97). Students focused on peers and highly value the opinions of their peers (Meschke et al., 2012). This implied that students who work with others who value academics would then begin to see the importance of academics. Two important studies, Dogru (2013) and Dobao (2014), further explored the benefits of students working in collaborative groupings.

Dogru (2013) focused on the effects of peer instruction on students' motivation and success when learning science at a middle school level. The purpose of this study was to identify whether there was a difference in motivation and in the amount of increased success on Academic Success Test Scores between students who participated in peer teaching as compared to those that participated only in traditional methods of instruction. The participants were two groups of seventh grade students (total 47 students) in Antalya, Turkey. The findings indicated that there was no statistically significant difference between the experiment group and the control group when motivation was measured. However, there was an increase in the post-test scores mean only for the students who received peer instruction, but not for the students who received traditional instruction. There was a statistically significant difference between the two groups on the Academic Success Test Scores, suggesting that students that were given peer instruction benefit greatly from this alternate method of instruction (Dogru, 2013).

Dobao (2014) focused on the advantages of pair and group collaborative work while learning vocabulary and language use in a second language. The study followed 110 Spanish as a second language students from a public university in the US. The study used pre- and post-test data from a vocabulary task and an individual writing task to assess student learning. The researchers also collected data from classroom group observations. Findings revealed that students reflected on and corrected their language use more in groups than in pairs; however, the opportunity for this reflection was still present in pairs. The researcher found that students benefited not only from their own interactions with peers and with the material, but also from observing interactions and collaborative problem-solving of peers in their group. Collaborative groups larger than two indicated more resources and more possible collaborative peer interactions (Dobao, 2014). As we can see from these studies, collaborative groups and peer instruction can facilitate learning for adolescents in the classroom.

Social Constructivism in the Mathematics Classroom.

As discussed in the Theoretical Rationale in Chapter 1, social constructivism aims to help students build their own understanding of concepts as they interact with their peers and teacher. Students can only do so much alone, but are able stretch their abilities and work within their zone of proximal development (ZPD) when they have the aid of a teacher or peer. Glaserfeld (1989) is quoted in Grady, Watkins, and Montalvo (2012) when he identifies one of the primary principals of constructivism to be "knowledge is not passively received but actively built up by the cognizing subject" (p. 39). Social constructivism, however, does not rely on the student to do all of the work. The teacher must be involved in carefully directing student learning.

The teacher's role in the classroom is to be a guide, a facilitator, and to have clear expectations and communicate those to students. In Goos (2004), the participating teacher in the study had a solid understanding of both mathematics and the concept of the zone of proximal development. His actions were justified by these theories and he also actively reflects on how his actions affected the students. Goos organized the teacher's actions into nine categories as follows:

1. The teacher models mathematical thinking.
2. The teacher asks students to clarify, elaborate, and justify their responses and strategies.
3. The teacher emphasizes sense-making.
4. The teacher makes explicit reference to mathematical conventions and symbolism.
5. The teacher encourages reflection, self-monitoring, and self-checking.
6. The teacher uses the students' ideas as starting points for discussion.
7. The teacher structures students' thinking.
8. The teacher encourages exploratory discussion.
9. The teacher structures students' social interactions (Goos, 2004, p. 267).

The participating teacher in the research performed by Goos (2004) had an end goal for his students and used these actions in order to reach that goal. His students did not blindly stumble through the mathematics. The mathematics lessons were not entirely constructed by the students, but rather the teacher had clear objectives for each lesson and guided the students through their understanding. "The teacher scaffolded the students' thinking by providing a predictable structure for inquiry through which he enacted his expectations regarding sense-making, ownership, self-monitoring, and justification" (Goos, 2004, p. 282). The teacher modeled inquiry and questioning for his students to provide a framework for them when they explored in their peer groups.

Successful teachers have a positive relationship with students their students and can help them to take more responsibility in the classroom. When discussing the study, Goos (2004) commented that, “as the school year progressed, the teacher gradually withdrew his support to pull students forward into more independent engagement with mathematical ideas” (p. 282). Students must be able to have a positive relationship with their teacher so that they can trust when the teacher moves them into an activity or role that might be slightly uncomfortable at first. An excellent teacher knows when to stretch their students and how to gradually release them into a more independent venture.

According to Lerman (2001), “learning mathematics or learning to think mathematically is learning to speak mathematically” (p. 107). Talking about a topic helps students to grapple with their actual understanding of the topic. If they are able to put their understanding into words, then they have a conceptual understanding of the topic. Once they voice their understanding, they are able to get feedback from their peers and teacher. Goos, Galbraith, and Renshaw (2002) discuss the benefits of collaborative group work. They introduce the idea of a “collaborative ZPD” that can be obtained when students with a similar level of understanding are working together on a mathematics problem. The study sought to find the differences between successful group work and unsuccessful group work, in which the group did not confidently arrive at an agreed-upon answer. The researchers found that students needed to be participating in both metacognition and transactive discussions in order to be successful. When students are participating in metacognition, they may be coming up with new ideas, analyzing the validity of an idea, or executing a certain strategy. Transactive discussions are defined as students presenting ideas to one another, asking questions, and discussing the validity of their peers' ideas. “The small group format for mathematical problem solving does seem to promote spontaneous verbalization, which allows students to offer their ideas for critical examinations” (p. 197). The researchers found that when students used an incorrect strategy or arrived at a wrong answer, peers were able to catch this or the students themselves was able to recognize their mistake by attempting to justify their answer.

In an interactive classroom, students are exploring, learning, and also teaching. Because students are conversing with one another in order to grapple with the meaning of the mathematics, students can become a teacher in their peer group. In the study by Goos (2004), students were able to help their peers develop an understanding of the concepts through some of the same techniques employed by the teacher. “This reorganization of classroom social interactions is crucial to understanding that the creation of the ZPD is a process of negotiated personal meanings and comparing these with conventional interpretations from the community of mathematics” (p. 282).

There are interactive classrooms in which students are being asked to communicate about mathematics and in which teachers value mathematical thinking, not just computation. Ellis and Berry (2005) refer to studies that have found that students in interactive classrooms outperform those who are receiving more traditional education in both computational mathematics and mathematical thinking and application. In addition, these students were more interested and motivated about mathematics.

The constructivist movement has not been widely accepted for several reasons. Student performance was measured by scores on standardized tests. Measuring student knowledge in terms of deep understanding is difficult to quantify on a standardized performance test.

Teachers adjusted instruction to fit the assessment. In fact, today schools are unable to fully commit to a curriculum or instructional direction because the Smarter Balanced Assessment that tests CCSS is still being changed. Because these tests are so high-stakes, educators are not going to take a chance on alternate methods of instruction, even if it leads to deeper understanding, when their current methods of instruction are giving them high test scores.

Summary

The research literature showed that mathematics teachers followed a pattern of instruction throughout the years, which focused on the transmission of knowledge. This tradition continued for years with no real changes in instructional practice and with limited attention to how teachers teach mathematics. Political leaders and educators only focused on changing mathematics practices when the US experienced a political or economic decline.

In the teacher/researcher's experience as a mathematics teacher, it appeared that many students were bored and perhaps only learning mathematics on a superficial level. This caused the teacher/researcher to reflect on her teaching practice and her knowledge of the emerging brain research.

Similar to the studies by Lau (2005) and Goos (2004), this study focused on implementing educational practices that promoted socialization and dialogue between the teacher and the students as well as among students. In the design and implementation of the study, the researcher considered the developmentally appropriate practices based on brain research. The research in the literature review points to the idea that dialogue and interaction may lead to a deeper understanding of mathematics.

Chapter 3 Method

Research Approach

This was a participatory teacher action research study. This means that I, as the teacher/researcher, am the researcher and conducted the study with the purpose of improving practice. In participatory teacher action research studies, the teacher recognizes that students are not developing a deep understanding and appreciation of the subject, in this case mathematics. The teacher then creates lesson plans and instruction and evaluates student performance over time. The students were also participating in the process—they were aware of the intention of the pre-and post-tests, knowing that they would compare the tests (Hendricks, 2013).

The participants were a group of eighteen 7th, and 8th graders from lower and middle class families. The evaluation research study used mixed methods. There was qualitative data from pre- and post-test scores as well as quantitative data from informal teacher/researcher observation. The evaluation research looked at student pre- and post-test scores using a standard chapter test on number sense and algebra. The scores were compared following teaching practices involving teacher-student and student-student interaction. The purpose was the improve student understanding and teacher practice.

Ethical Standards

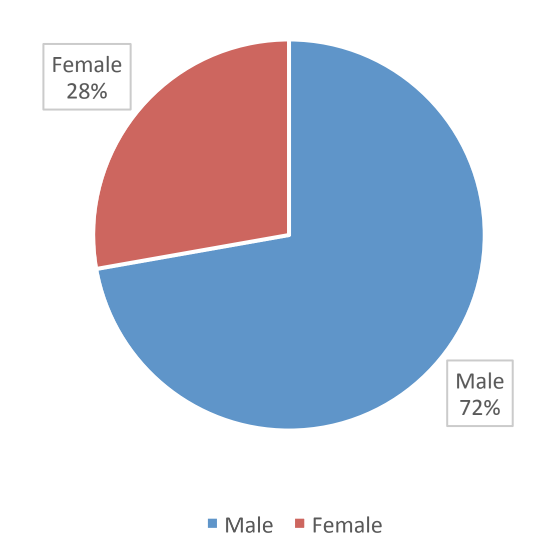
This paper adheres to the ethical standards for protection of human subjects of the American Psychological Association (2010). Additionally a research proposal was submitted and reviewed by the Dominican University of California Institutional Review Board for the Protection of Human Subjects (IRBPHS), approved and assigned number 10372.

Sample and Site

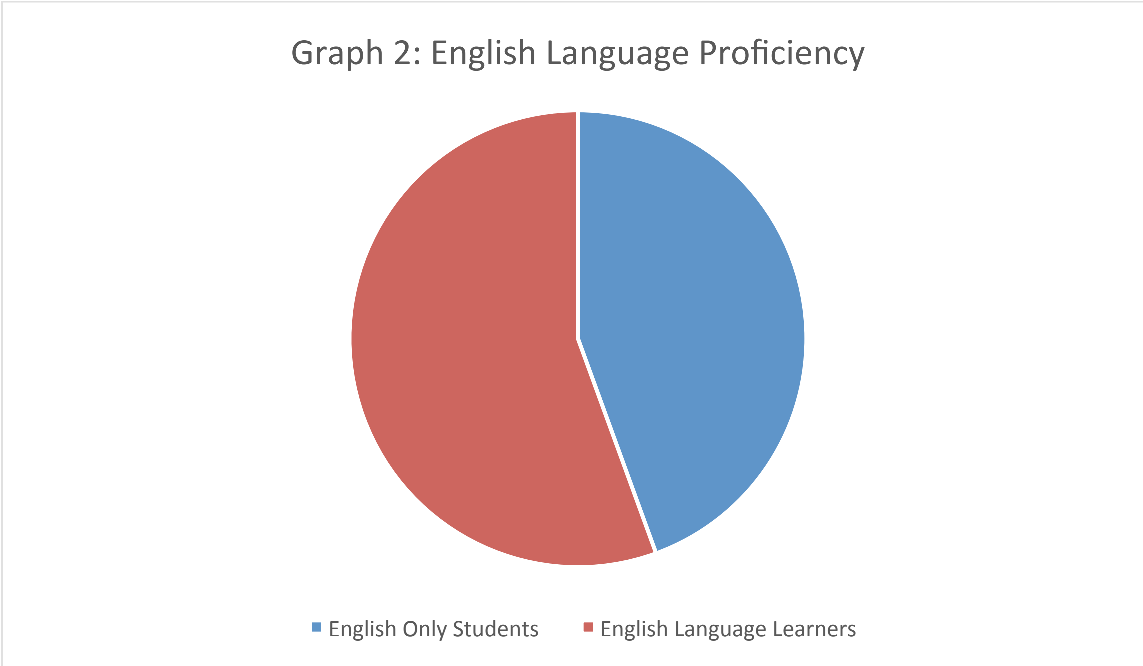
The school site where the study took place is a small charter and public school. At the time of the study, there were a total of 271 students that attended the school, 143 male students (53%) and 128 female students (47%). A large number of students received free or reduced lunch, a total of 71% of the school. Many of the families speak Spanish only at home and as a result, 57% of the students are English language learners.

The participants were a group of 18 middle school students in 7th and 8th grade. They are in a class where they are learning higher level 8th grade material, so only students who have mastered the 7th grade standards are placed in the class. Six of the eighteen students were placed in the class after the first unit was covered once it was clear that they were ready for more advanced material. As a result, these 6 students only participated in the second half of the study.

Graph 1: Gender of Participants



Graph 1 shows the percentage of male and female students who participated in the study. There were a large number of male participants.



Graph 2 shows that the percent of English Language Learners (ELLs) and English Only students (EOs) was balanced in this study.

Table 1: Age of Students

Age	12	13	14
Number of Students	2	12	4

Table 1 above shows the age distribution of the students in the study.

Access and Permissions

The teacher/researcher was the credentialed teacher of record and these students in 7th and 8th grade were assigned to the teacher/researcher. Evaluating and improving teaching practices is a part of the teacher/researcher's job as a professional.

Data Gathering Procedures

The research began on August 19, 2015 and ended on October 9, 2015. For at least 30 minutes during an 80 minute period each day of instruction, students collaborated with one another to solve problems that were in their ZPD. These mathematics challenges either required students to use skillsets that were taught during the previous class or related to the objective on that day. During this collaboration time, students asked each other questions about the material and justified their methods. The teacher/researcher interacted with students to lead them toward a deep understanding of number sense and algebra. The teacher questioned and probed, perhaps pointing out misconceptions or flaws in their students' logic and suggesting alternative approaches to problems. Students also participated in a traditional session during the period in which they took notes from a lecture and then did independent practice.

The teacher/researcher took note of how promoting interaction surrounding the mathematics affected change in the students. At first, the teacher/researcher gave students simple sentence starters and the students practiced using them. Examples of the sentence starters are: "Can you help me understand if my idea makes sense?"; "Good thinking, but I have another idea."; "Have you thought of…?" Students practiced these formally during one class period and were reminded to use them informally by the teacher/researcher during group work time.

When asked to reflect on the benefits and drawbacks of group work, the students responded that collaboration helps them to accumulate more ideas. They noted that they are able to brainstorm and interact. They commented that it helps them to actually form and voice their own opinions about the mathematics.

There were two unit tests that were used for the study. The same test was given as both the pre- and post-test. No information is available on the validity or reliability of the tests.

Data Analysis Approach

The scores of the pre- and post-tests were compared and analyzed to evaluate the effectiveness of those strategies on the participants' understanding of mathematics. The percent correct on each pre- and post-test were put into a spreadsheet so that the increase in percent correct could be noted. In addition to this quantitative data, the teacher/researcher looked for patterns that emerged from the teacher/researcher narrative and the student narrative.

Chapter 4 Findings

Results

The teacher/researcher sought to find whether increased social interaction and dialogue with other students and the instructor increases student understanding of number sense and algebra. All students in the study increased their understanding of algebra and number sense as evidenced by their test scores and teacher observation. The research data showed that students experience increased understanding of mathematics after participating in social interaction and dialogue.

When analyzing the teacher and student narratives, the teacher/researcher noted that the students demonstrated more comfort in working with peers as the study progressed. The students appeared to work well on simpler tasks together even at the beginning. However, when the tasks increased in difficulty, some students tended to resort to working alone. As they became more comfortable with working together on both straight-forward and challenging tasks, they chose it more often. After a few weeks, students started using phrases that the teacher/researcher introduced to them as appropriate ways to interact. They also used similar phrases without prompting and would ask for peer feedback and critique on their own. “Did you get this answer?”; “Wait, how did you come up with that?”; “Can you explain it to me?”; “I am not sure that is correct because…” were some of the interactions that the teacher/researcher heard. Overall, students began dialoging on their own to share their answers with others or ask for a peer to evaluate the validity of an idea.

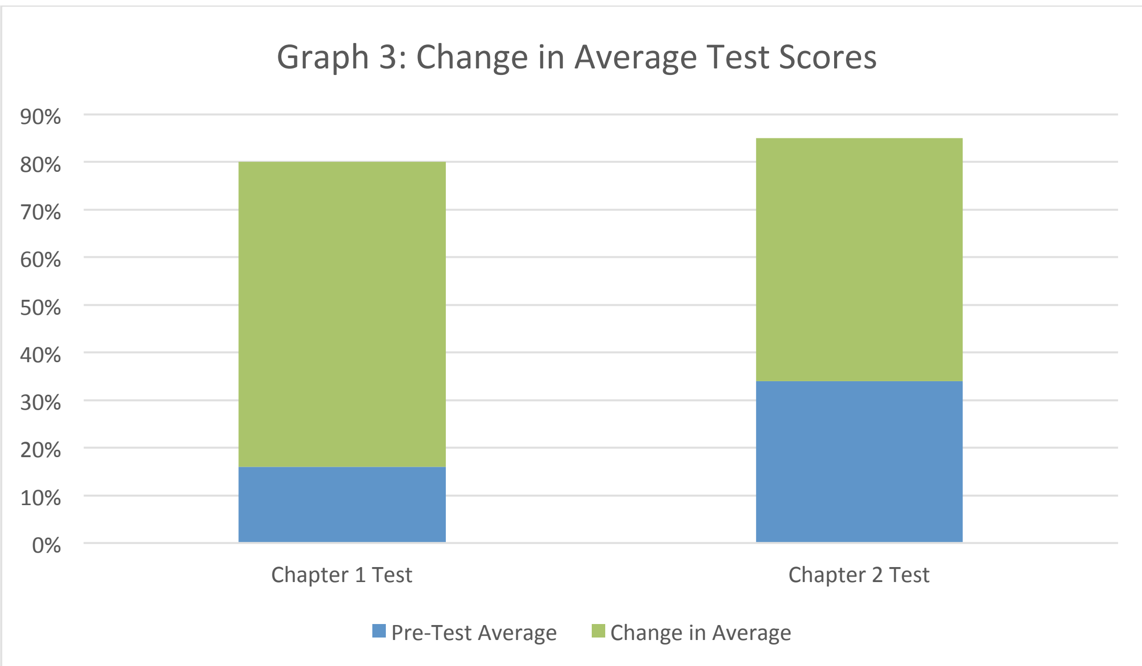
The results from the pre- and post-tests showed an increase in all individual and mean scores, as shown in the Table 2, Table 3, Graph 3, Graph 4, and Graph 5 below.

Table 2

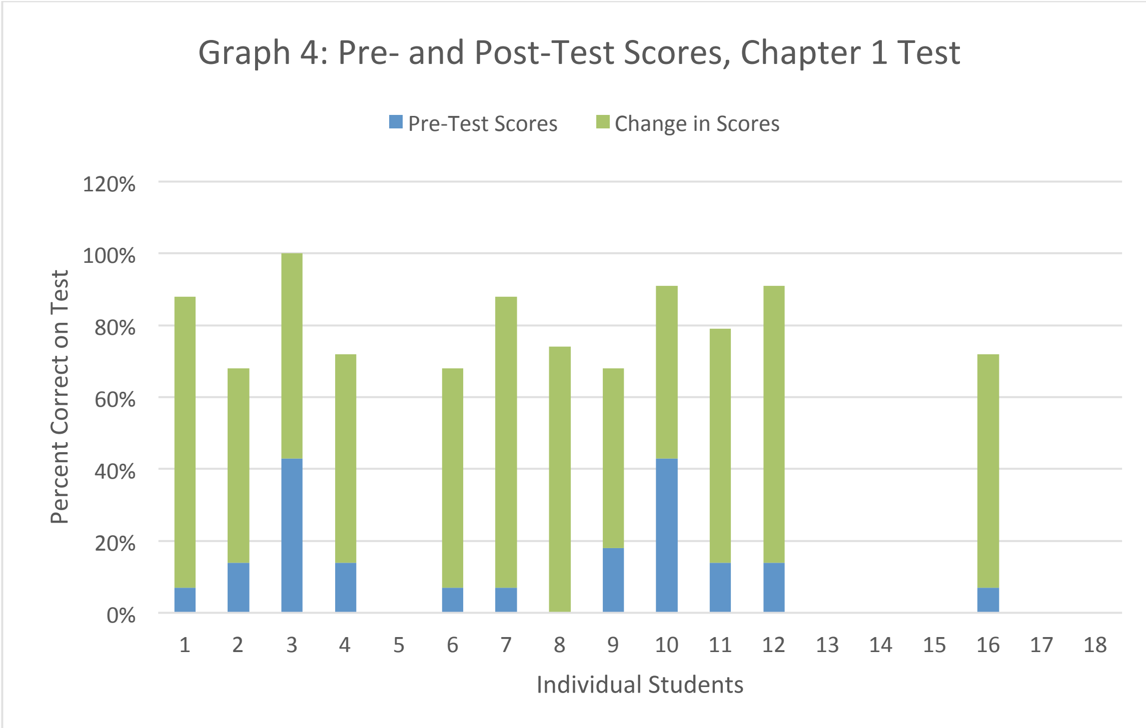
Group A, Pre-test 1 Mean Scores	Group A, Post-test 1 Mean Scores	Change in Mean Scores
16%	80%	64%

Table 3

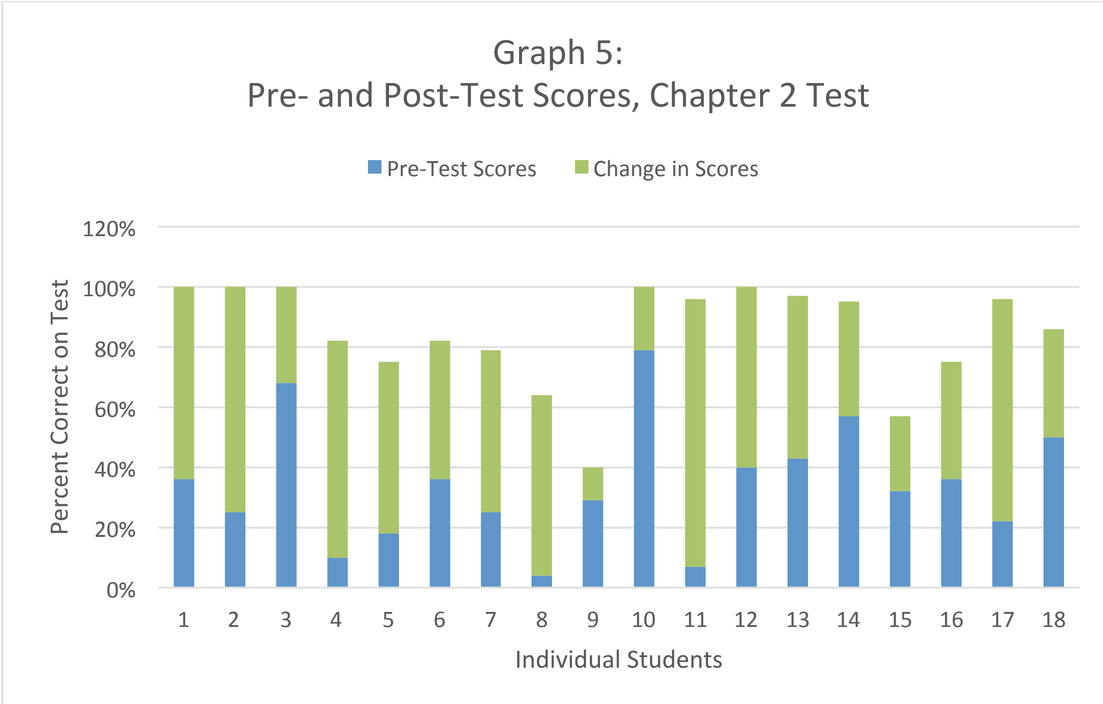
Group A, Pre-test 2 Mean Scores	Group A, Post-test 2 Mean Scores	Change in Mean Scores
34%	85%	51%



These results indicate that students increased their understanding of number sense and algebra after experiencing effective teaching strategies.



The chart above shows the pre- and post-test scores for the Chapter 1 assessment for each student that participated in the study. The change in scores indicates growth of understanding of number sense. Students who do not have scores did not participate in the first half of the study. They joined the class in time to participate in the Chapter 2 pre- and post-test only.



The chart above shows the pre- and post-test scores on the Chapter 2 assessment for each student that participated in the study. The change in scores indicates growth of understanding of algebra.

Summary

The data from the pre- and post-tests showed that all students showed an increase in understanding of both number theory and algebra. Students' comfort surrounding group work, dialogue, and interaction increased throughout the study, as noted in both the teacher and student reflections. In conclusion, all students in the study demonstrated an increase in understanding of number sense and algebra after experiencing interaction and dialogue in the mathematics classroom.

Chapter 5 Discussion /Analysis

Summary of Major Findings

The primary research question for this study was: How does increased social interaction and dialogue with other students and the instructor increase student understanding of number sense and algebra? The findings showed that all students increased their understanding of number sense and algebra. However, not all students increased their understanding to the point of mastery. One of the conclusions that can be drawn from the study is that students benefit from interaction with their peers and the teacher when discussing mathematical content.

Comparison of Findings to the Literature

CCSS (Common Core State Standards Initiative, 2015) and the low US PISA scores (OECD, 2015a) call for a deep understanding of mathematics. This implies that educators need to adapt their teaching practice and provide opportunities for students to engage in active problem-solving and critical thinking. The teacher/researcher considered this and adjusted their teaching practices to include strategies that promoted student interaction. The pre- and post-test score comparisons demonstrated that students benefited from these interactions. This implies that students may perform better on the Smarter Balanced Assessment, the standardized test for CCSS.

The current study was based upon research findings concerning best practices for mathematics education of adolescents. The teacher/researcher considered these when developing the study and found that the practices employed were beneficial for student understanding of mathematics for students in grades 7 and 8. In many studies (Dobao, 2014; Goos, 2004; Lau,

2009) researchers found success in a model that featured peer interaction as a key element. The present study supported the idea that peer interaction and dialogue about mathematics concepts increases student understanding of mathematics concepts such as number sense and algebra.

Limitations/Gaps in the Research

This sample consisted of a small study from only one school site. The participants were a small group of math students who were placed in this math class in order to best serve them. The mathematics that was taught in this class was at a higher level than other classes taught at the school. This was a participatory teacher action research study, so the teacher/researcher had direct influence on the research and the attitudes of the participants. There was limited time, so just two mathematics units with pre- and post-tests were given.

There is no way of knowing which instructional strategy actually promoted growth in understanding for the students. Social interaction was employed as a strategy, but so were traditional methods such as lecturing and practicing sets of problems individually. In this study, there was not a group of students (used as a control group) that did not experience social interaction and dialogue. Therefore, the results should not be generalized until further research is conducted.

Overall Significance of the Study

This study explored how students can reach a deep understanding of mathematics. Additionally, this study allowed the teacher/researcher to evaluate their practices in a structured way and analyze the outcomes of using interaction and dialogue in the classroom. The results imply that socialization and dialogue with peers and teachers about mathematics concepts increases student understanding of those concepts. This research added to the literature concerning best practices in math education for adolescents. It can be a first step toward future research that explores the benefits of social interaction and dialogue for middle school students in the mathematics classroom.

Implications for Future Research

The study implies that mathematics instructors should employ social interaction and dialoguing about mathematics in the mathematics classroom. In order for change to occur in mathematics classrooms, teachers need specific professional development on how to incorporate these practices. To extend this study, more students in schools with varying demographics could be tested before and after receiving instruction that involves peer and teacher interaction. A future study could research how student self-perception of their abilities in mathematics changes after these strategies are employed in their mathematics classroom. Also, future studies could employ control groups so that more conclusions could be drawn about the effectiveness of applying interaction and dialogue as an instructional strategy. Longitudinal studies could track students' extended achievement and engagement in math.

About the Author

Catherine Janes is a middle school math, science, and art teacher. Teaching and mathematics have been her passions since she was in middle school herself. She graduated from Scripps College with her B.A. in mathematics in 2010. She obtained her multiple subject and single subject mathematics teaching credential from Dominican University of California in 2011. As a former mathematics major, she is interested in making mathematics more engaging for her students.

References

American Psychological Association. (2010). *Publication manual of the American Psychological Association*. Washington, DC: American Psychological Association.

Beckmann, S. (2010). From the Common Core to a community of all mathematics teachers. Mathematics Educator, 20(2), 3-9. Retrieved from http://search.ebscohost.com

Blakemore, S. (2007). Brain development during adolescence. *Education Review, 20*(1), 82-90. Retrieved from http://search.ebscohost.com

Brown, T. (2009). An exploratory study of mathematics engagement of secondary students. (Doctoral dissertation). Retrieved October 31, 2015 from http://scholarworks.gsu.edu/msit_diss/47

Casey, B. J., Tottenham, N., Liston, C., & Durston, S. (2005). Imaging the developing brain: What have we learned about cognitive development? *Trends in Cognitive Sciences, 9*(3), 104-110. doi:10.1016/j.tics.2005.01.011

Center for the Study of Mathematics Curriculum. (2004). Curriculum and evaluation standards for school mathematics: National Council of Teachers of mathematics Commission on standards for school mathematics 1989. Retrieved April 19, 2015 from http://www.mathcurriculumcenter.org/PDFS/CCM/summaries/standards_summary.pdf

Chen, I. (n.d.). *An electronic textbook on instructional technology*. Houston, TX: University of Houston, College of Education. Retrieved April 18, 2015 from http://viking.coe.uh.edu/~ichen/ebook/et-it/constr.htm

Choudhury, S., Charman, T., & Blakemore, S. (2008). Development of the teenage brain. *Mind, Brain & Education, 2*(3), 142-147. doi:10.1111/j.1751-228X.2008.00045.x

Common Core State Standards Initiative. (2015). Key Shifts in Mathematics. Retrieved October 31, 2015 from http://www.corestandards.org/other-resources/key-shifts-in-mathematics/

Dobao, A. F. (2014). Vocabulary learning in collaborative tasks: A comparison of pair and small group work. *Language Teaching Research, 18*(4), 497-520. doi:10.1177/1362168813519730

Dogru, M. (2013). The effects of peer instruction on the success, motivation and decision-making styles of primary seventh grade students. *International Journal of Academic Research, 5*(5), 299-304. doi:10.7813/2075-4124.2013/5-5/B.46

Ellis, M. W., & Berry, R. Q. (2005). The paradigm shift in mathematics education: Explanations and implications of reforming concepts of teaching and learning. *Mathematics Educator, 15*(1), 7-17. Retrieved from http://search.ebscohost.com

Goos, M. (2004). Learning mathematics in a classroom community of inquiry. *Journal for Research in mathematics Education, 35*(4), 258-291. Retrieved from http://www.jstor.org/stable/30034810

Goos, M., Galbraith, P., & Renshaw, P. (2002). Socially mediated metacognition: Creating collaborative zones of proximal development in small group problem solving. *Educational Studies in Mathematics, 49*(2), 193-223. Retrieved from http://search.ebscohost.com

Grady, M., Watkins, S., & Montalvo, G. (2012). The effect of constructivist mathematics on achievement in rural schools. *Rural Educator, 33*(3), 38-47. Retrieved from http://search.ebscohost.com

Hendricks, C. (2013). Improving schools through action research: A reflective practice approach (3rd ed.). San Francisco, CA: Pearson.

Lau, P. N., Singh, P., & Hwa, T. (2009). Constructing mathematics in an interactive classroom context. *Educational Studies in Mathematics, 72*(3), 307-324. doi:10.1007/s10649-009-9196-y

Leistyna, P. (2004). Presence of mind in the process of learning and knowing: A dialogue with Paulo Freire. *Teacher Education Quarterly, 31*(1), 17-29. Retrieved from http://search.ebscohost.com

Lerman, S. (2001). Cultural, discursive psychology: A sociocultural approach to studying the teaching and learning of mathematics. *Educational Studies in Mathematics, 46*(1), 87-113. Retrieved from http://search.ebscohost.com

Lockhart, P. (2009). *A mathematician's lament*. New York, NY: Bellevue Literary Press.

Meschke, L., Peter, C., & Bartholomae, S. (2012). Developmentally appropriate practice to promote healthy adolescent development: Integrating research and practice. *Child & Youth Care Forum, 41*(1), 89-108. doi:10.1007/s10566-011-9153-7

Organisation for Economic Co-operation and Development (OECD). (2015a). International student assessment (PISA). Retrieved on April 19, 2015 from https://data.oecd.org/pisa/mathematics-performance-pisa.htm

Organisation for Economic Co-operation and Development (OECD). (2015b). Programme for international student assessment (PISA) results from PISA 2012: United States. Retrieved on April 9, 2015 from http://www.oecd.org/unitedstates/PISA-2012-results-US.pdf

Pöhland, L., & Raufelder, D. (2014). The role of peers and teachers as motivators on adolescents' neural emotional processing predicting feelings of loneliness, depression and stress: Results of an interdisciplinary study. Diskurs Kindheits- Und Jugendforschung, 9(4), 451-465.

Sercombe, H., & Paus, T. (2009). The 'teen brain' research: An introduction and implications for practitioners. *Youth & Policy, 103*(Summer), 25-37. Retrieved from http://search.ebscohost.com

Southwest Educational Development Laboratory (SELD). (1994). Constructing knowledge in the classroom. *Classroom Compass, 1*(3), Retrieved March 9, 2015 from http://www.sedl.org/scimath/compass/v01n03/1.html

Steele, A., Brew, C., Rees, C., & Ibrahim-Khan, S. (2013). Our practice, their readiness: Teacher educators collaborate to explore and improve preservice teacher readiness for science and math instruction. Journal of Science Teacher Education, 24(1), 111-131. doi:10.1007/s10972-012-9311-2

U.S. Department of Education. (2008). A nation accountable: Twenty-five years after "A nation at risk". Washington, D.C.: US Department of Education. Retrieved from https://www2.ed.gov/rschstat/research/pubs/accountable/accountable.pdf

Yavuz Mumcu, H., & Cansiz Aktas, M. (2015). Multi-program high school students' attitudes and self-efficacy perceptions toward mathematics. Eurasian Journal of Educational Research, 59, 207-226 http://dx.doi.org/10.14689/ejer.2015.59.12

Appendix

Letter to Principal Requesting Permission to Conduct the Study

Dear Superintendent/Principal,

I am currently enrolled in a master's degree of education at Dominican University of California, San Rafael, CA. The purpose of my study is to evaluate the effectiveness of student interaction and dialogue surrounding mathematics concepts. I am requesting your consent for me to evaluate students' pre- and post-test score data on 7th and 8th grade math.

My research project took place in the fall of 2015. As the classroom teacher, I created tests to evaluate student progress in developing the concept of number sense and algebra. Additionally I took notes on comments students made that reflected their discussion and understanding of number sense in alignment with Common Core Standards for Mathematics. Results of the study were developed in summary form only. No personally identifiable information is included in the final report. At the conclusion of the study, once the paper receives final departmental approval, the thesis will become part of the Dominican University of California repository, Dominican Scholar.

This research project has been reviewed by my faculty advisor and approved. If you have any questions, please do not hesitate to contact me or, if you prefer, Dr. Madalienne Peters at Dominican University of California, (415) 484-3285.

Sincerely,
Catherine Janes

I agree that Catherine Janes may collect information on student performance related to increasing number sense and algebra as part of her research for the degree, Master of Science in Education. I also understand that the final thesis will become part of the research database, Dominican Scholar, Dominican University of California.

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Signature

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Date